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Power law velocity fluctuations due to inelastic collisions in numerically simulated vibrated bed of powder

Y-h. Taguchi*
Department of Physics, Tokyo Institute of Technology
Oh-okayama, Meguro-ku, Tokyo 152, Japan

Hideki Takayasu
Graduate School of Information Sciences
Tohoku University, Sendai 980-77, Japan

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Abstract

Distribution functions of relative velocities among particles in a vibrated bed of powder are studied both numerically and theoretically. In the solid phase where granular particles remain near their local stable states, the probability distribution is Gaussian. On the other hand, in the fluidized phase, where the particles can exchange their positions, the distribution clearly deviates from Gaussian. This is interpreted with two analogies; aggregation processes and soft-to-hard turbulence transition in thermal convection. The non-Gaussian distribution is well-approximated by the t-distribution which is derived theoretically by considering the effect of clustering by inelastic collisions in the former analogy.

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Granular matter are attracting much interest of physicists [1]. Among them, people extensively study vibrated bed of powder, which consists of a vessel filled with granular matter and a loud speaker to shake it. When the acceleration amplitude of the vibration Γ, the control parameter, exceeds some critical value, the bed is fluidized and can show many interesting phenomena: heaping, convection [2], capillarity [3], surface fluidization [4,5], Brazil nuts segregation [6–10], standing waves [11–14], and so on.

In this letter we first report that dynamical phase transition takes place in numerically simulated vibrated bed of powder: probability distribution function (PDF) of displacement vector changes from Gaussian to non-Gaussian. This phase transition is interpreted in two ways: a soft-to-hard turbulence transition similar to that of fluid turbulence and the averaging effect of particle velocities due to inelastic clustering of granules.

The numerical setup (2D) used in this letter has been described in detail in Refs. [15]. Numerical simulations not only reproduce convection [16,17], but also show powder turbulence [15,18]. The particles interact by visco-elastic collisions [19], and their motions are integrated under the existence of gravity acceleration g. The material parameters are the collision time t_{col}, the time period during collision, and the coefficient of restitution e. The bed is two-dimensional, with a periodic condition along the horizontal direction. The bottom of the vessel oscillates vertically as a function of time $t, b\cos\omega_0 t$, where $\Gamma = b\omega_0^2$.

In order to keep all of the particles in a fixed area, a lid (or a weight) is put horizontally on the granular layer. The spacing between the lid and the bottom fluctuates as particles hit the lid. This spacing can be used as a guide to observe stationary statistics; namely, we observe the locations of the particles at a time when the spacing takes a value in a given small interval $[h_0, h_0 + dh_0]$. Let the nth observation time be $t(n)$ and the ith particle's location at time $t(n)$ be $\mathbf{x}_i(n)$, then the relative velocity $\tilde{\mathbf{v}}_i(n)$ is defined by the displacement $\Delta\mathbf{x}_i(n) = \mathbf{x}_i(n) - \mathbf{x}_i(n-1)$ over the time interval $\Delta t(n) = t(n) - t(n-1)$.

Our numerical simulation is performed with the following parameters: $g = 1.0, \omega_0 = 2\pi/6, b = 1.0(\Gamma = 1.1)$, the total number of particles $N_{tot} = 1024$, the horizontal width of the bed $L_h = 128$, the particle diameter $d = 2.0$, the particle mass $m = 1, t_{col} = 0.1, e = 0.8$, the mass of the lid $M = 100$, and $h_0 = 30.0$. With these parameters we can find two phases coexisting in the vessel: the lower region of the layer belongs to the solid phase in which no pair of particles exchange their position, and the upper region belongs to the fluidized phase where particles exchange their positions and show fluid-like collective motions. These two phases both have the Kolmogorov's power spectrum, $k^{-5/3}$ [15], where k denotes the wave number for the relative velocity field $\tilde{\mathbf{v}}_i^{(n)}$. Both phases are thus regarded as turbulent states. However, they have completely different PDFs ; the PDF in the solid region is close to Gaussian, but the fluidized region has a PDF very different from Gaussian(Fig. 1). For larger Γ, ($\Gamma = 1.64$: b is changed to be 1.5 from $b = 1.0$ when $\Gamma = 1.1$) the bed is now fully fluidized and we can find non-Gaussian PDFs (Fig. 2).

In order to understand this phenomenon, we try to interpret it in two ways. First analogy is aggregation processes of inelastically colliding particles. Actually speaking, it recently turns out that inelastically colliding particles can exhibit non-Gaussian PDF of velocity [20]. Thus this analogy seems to be suitable for understanding this phenomenon. In order to explain the non-Gaussian tails theoretically, let us assume that N particles having velocities $\{v_i\}$ exchange their momenta frequently through mutual inelastic collisions. By the conservation of total momenta it is obvious that each velocity v_i gets closer to the

averaged velocity, $V = 1/N \sum v_i$, as inelastic collisions are repeated. Assuming also that the velocity distributions for v_i before collisions follow independent identical Gaussian with the mean value $v = 0$ and variance σ^2. Then the distribution of the averaged velocity V is given as

$$P_N(V) = \frac{1}{\sqrt{2\pi\sigma^2/N}} \exp\left(-\frac{V^2}{2\sigma^2/N}\right). \tag{1}$$

For a fixed value of N this PDF is also Gaussian, however, the value of N, which can be regarded as the size of a cluster,should be a random number. As a kind of mean-field approximation we assume that any pair of particles belong to the same cluster with an independent and identical probability. With this assumption the probability that a particle is a component of cluster of size N, $W(N)$, is given by an exponential function as, $W(N) = c\ \exp(-cN)$, where c is a positive constant. Then, the probability of finding a particle with velocity v after collisions is given as

$$P(v) = \sum_N W(N) \frac{1}{\sqrt{2\pi\sigma^2/N}} \exp\left(-\frac{v^2}{2\sigma^2/N}\right). \tag{2}$$

Approximating the summation in Eq.(2) by the integral $\int dN$ we get the following functional form:

$$P(v) = \frac{1}{2a}\left[1 + \left(\frac{v}{a}\right)^2\right]^{-3/2}, \tag{3}$$

where $a = \sigma\sqrt{2c}$. Note that this distribution is close to Gaussian in the vicinity of $v = 0$, however, for large $|v|$ it has power law tails of $|v|^{-3}$. In Fig. 2 this theoretical estimate of PDF [21] is compared with the PDF in the hard turbulent phase of our numerical experiments. We can find a good fit in the whole range of v, which validates our clustering assumption.

From a mathematical view point the PDF in Eq. (2) is a special case of a t-distribution. A general t-distribution can be derived by considering the following generalized cluster distribution as

$$W(N) \sim N^{\alpha-1} \exp(-cN). \tag{4}$$

For $1 \geq \alpha > 0$ the distribution is a decreasing function like the exponential distribution ($\alpha = 1$); however, for $\alpha > 1$ the distribution has a maximum around α/c. The particle velocity distribution with the general cluster distribution Eq.(4) becomes

$$P(v) \sim [1 + (v/a)^2]^{-\alpha-1/2} \tag{5}$$

This is the general form of the t-distribution. This PDF converges to a Gaussian in the limit of $\alpha \to \infty$, which corresponds to an infinite cluster size. For any finite α, $p(v)$ is close to Gaussian around $v = 0$, however, there always exist long tails in the power law.

The second analogy is thermal convection turbulence. Thermal convective turbulence is categorized into two phases according to their PDF [22]. In cases where the temperature difference between the upper and the lower boundaries of a fluid container is intermediate, we

can find the so-called soft turbulent phase, in which thermal convective flows show irregular fluctuations following Gaussian distributions. When the temperature difference is enlarged, another phase, the so-called hard turbulent phase, appears; in this phase the fluctuation clearly deviates from Gaussian showing long tails. Although much effort has been made to understand this phenomenon, the underlying mechanism of producing the non-Gaussian fluctuations has yet to be elucidated [23].

In our granular bed, it has already numerically been confirmed that both solid and fluid phase exhibits Kolmogorov's -5/3 powder spectrum [15]. Thus it is not a bad analogy to compare turbulent vibrated bed with fluid turbulence. If this analogy is acceptable, we may be able to get new insight about soft-to-hard phase transition. Compared with the thermal convection, the vibrated bed of powder has some advantages for study. First, our powder system has a finite number of degrees of freedom as compared with the infinite number in thermal convection. This finiteness can make the numerical simulation much easier; for example, a low performance personal computer can generate the hard turbulent state in the vibrated bed of powder. Second, observations in real experiments of powders are expected to be performed much more easily than those in fluid experiments. The displacement vectors in powder can be measured photographically, while the temperature or the velocity in the thermal convection can be measured only at one or a few fixed points using special devices. Clement and Rajchenbach [24] have already observed PDFs of displacement vectors in a fluidized region and concluded that PDFs are close to Gaussian. However, their observation is limited to the center of the distribution, and we believe that much better statistics are needed to observe the deviation from the Gaussian, since the deviation can be seen only when the long tails of the PDF are measured. In fact, recent experiments of vibrated bed have started to detect deviations from Gaussian PDF [25].

By joining these two analogies into one, it may be interesting to regard the soft turbulent phase (or the solid phase) as being composed of an infinite cluster. An understanding of the Gaussian velocity distribution in this phase follows from the discussion above. Also, the hard-turbulent phase (or the fluidized phase) may be regarded as the state that is composed of finite clusters. Therefore, it seems reasonable to view the soft-to-hard turbulence transition as a percolation-like phase transition. To put this view on a firmer foundation an intensive analysis of the processes of clusterings and momenta transports is required as a future task.

In conclusion we have found both theoretically and numerically that the velocity distribution in a vibrated bed of powder follows a Gaussian in the solid phase and it follows a t-distribution with power law tails of exponent -3 in the fluidized phase. Direct experimental confirmations of these results may be very promising. Also, a new theoretical approach to the soft-to-hard turbulence transition in thermal fluid convection may now be possible.

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* Electronic address: ytaguchi@cc.titech.ac.jp

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FIGURES

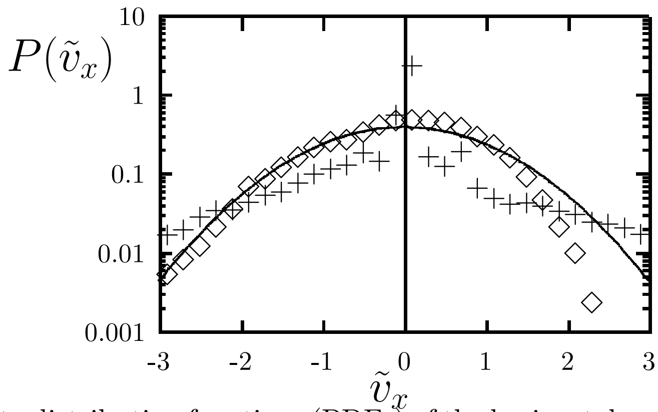

FIG. 1. Probability distribution functions (PDFs) of the horizontal component $\tilde{v}_x$ of the relative velocity $\tilde{\mathbf{v}}_i^{(n)}$. $+$: Fluidized region near surface (the region higher than the bottom by 10.0.) $\diamond$: Solid region below the fluidized region. Gaussian PDF is shown by the solid line for comparison. The abscissa is normalized by the standard deviation

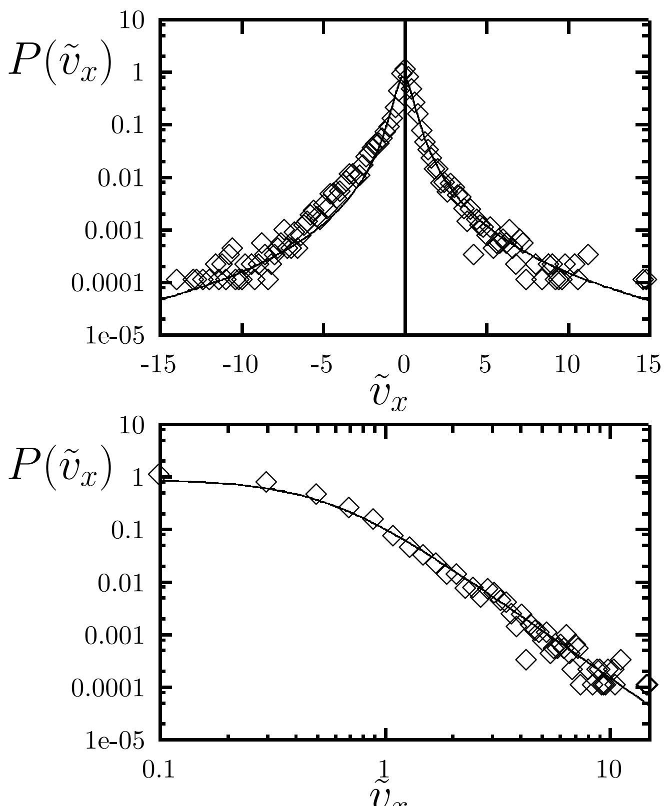

FIG. 2. The same probability distribution functions (PDF) as Fig.1, but for the fully fluidized bed (semi-log plot and log-log plot). The solid line shows the theoretical PDF of Eq.(3) with $a = 0.56$.